RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, JULY 2021
SECOND YEAR [BATCH 2019-21]

Date :08/07/2021 **MATHEMATICS**
Time : 11am-1pm **Paper** : MTM P 16 Full Marks : 50

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A

Integral Transform

Answer any four questions from question no. 1-6. [4 x 5]

1. State the formula of Laplace transform of the n-th derivative of a function. Hence evaluate $L\left[t^6\right]$, given that $L\left[t^3\right]=\frac{6}{p^4}$. [2+3]

2. Use the convolution theorem to evaluate $L^{-1}\left[\frac{p^2}{(p^2+a^2)^2}\right]$.

3. By applying Laplace transform, solve the following simultaneous ordinary differential equation.
$$\left.\begin{array}{r}(D-2)x(t)+3y(t)=0\\ 2x(t)+3(D-1)y(t)=0\end{array}\right\} t>0 \text{ where } D\equiv\frac{d}{dt}, \text{ subject to the conditions } x(0)=8, y(0)=3.$$

4. Find the complex Fourier transform of $F(x)=\begin{cases}(1-x^2), & |x|\le 1\\ 0, & x>1\end{cases}$ and hence evaluate $\displaystyle\int_0^\infty\left[\frac{x\cos x-\sin x}{x^3}\right]\cos\frac{x}{2}dx$.

5. Find the Fourier sine transform e^{-x} and hence prove that $\int_0^\infty \frac{t\sin tx}{1+t^2}dt = \frac{\pi}{2}e^{-x}$.

6. Solve $\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$, $x>0, t>0$ subject to the condition $u(0,t)=0$, $u(x,0) = \begin{cases} 1, & 0<x<1 \\ 0, & x>1 \end{cases}$; $u(x,t)$ is bounded.

Group B

Integral Equation

Answer any two questions from question no. 7 to 9. [2 x 7.5]

7. Solve the following Fredholm integral equation with the help of resolvent kernel:

$$y(x) = 1 + \lambda \int_0^1 (1-3xt)y(t)dt.$$

8. Find the value of λ for which the following integral equation has a non-trivial solution:

$$u(x) = \lambda \int_0^\pi K(x,t)u(t)dt; \; x \in \; [0,\pi]$$

where

$$K(x,t) = \begin{cases} \sin x \cos t, & \text{when } 0 \le x \le t \\ \sin t \cos x, & \text{when } t \le x \le \pi. \end{cases}$$

9. (a) Solve: $\phi(x) = \cos x + \lambda \int_0^\pi (\sin x)\, \phi(t)dt.$ [2.5]

   (b) Solve the following integro-differential equation : $g'(x) + 5\int_0^x (\cos 2(x-t))\, g(t)dt = 10.$ [5]

Group C

Calculus of Variation

Answer any two questions from question no. 10 to 12. [2 x 7.5]

10. (a) Find the extremal of the functional $\int_0^1 ((y')^2 + (z')^2 + 4z)dx$ that satisfy the boundary conditions $y(0)=0,\; y(1)=1,\; z(0)=0$ and $z(1)=0$. [5]

    (b) Find the second variation of the functional $\int_{x_1}^{x_2} (6y+79)dx$. [2.5]

11. (a) Find the Euler-Ostrogradsky equation for $I\left[z(x,y)\right] = \iint_D \left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2 + 2\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 - 2zf(x,y) \right] dxdy.$ [2.5]

    (b) Find the extremal of the functional $I\left[y(x)\right] = \int_{x_1}^{x_2} (y^2 + (y')^2 + 2ye^x)dx.$ [5]

12. Find the shortest distance between $y^2 = 4x$ and the straight line $x+y=-5$.

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